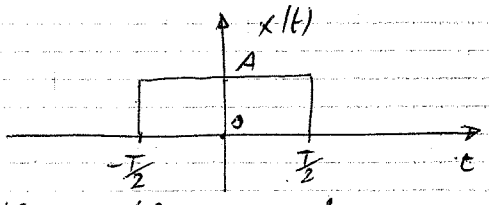
(Es_TCF_1_pdf)

Ex 1 — TCF di un rettangolo

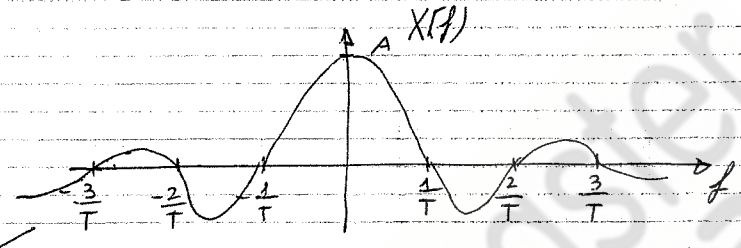
$x(t) = A\,\mathrm{rect}\left(\frac{t}{T}\right)$



$$x(t) = A\,\mathrm{rect}\left(\frac{t}{T}\right) \Leftrightarrow X(f) = AT\,\mathrm{sinc}(fT)$$

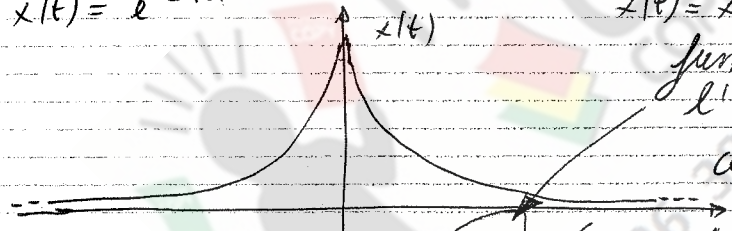
$$X(f) = \int_{-\infty}^{+\infty} x(t)\, e^{-j2\pi ft}\, dt = A\int_{-T/2}^{T/2} e^{-j2\pi ft}\, dt = A\cdot\left(\frac{1}{-j2\pi f}\right) e^{-j2\pi ft}\Big|_{-T/2}^{T/2} =$$

$$= \frac{A}{j2\pi f}\left(\frac{e^{j\pi f\frac{T}{2}(2)} - e^{-j\pi f\frac{T}{2}(2)}}{2j}\right) = \frac{A}{\pi f}\,\mathrm{sin}(\pi fT)\left(\frac{T}{T}\right) = AT\,\mathrm{sinc}(fT)$$



Ex 2 — TCF esponenziale bilatero

$x(t) = e^{-|t|}$



$x(t) = x(-t)$ è una funzione pari: posso calcolare l'integrale del I° quadrante.

con $|t| > 0 \Rightarrow t > 0$

$$X(f) = \int_{-\infty}^{+\infty} e^{-|t|}\, e^{-j2\pi ft}\, dt = \int_{-\infty}^{0} e^{t(1-j2\pi f)}\, dt + \int_{0}^{+\infty} e^{-t(1+j2\pi f)}\, dt =$$

$$\left(\frac{1}{1-j2\pi f}\right) e^{(1-j2\pi f)t}\Big|_{-\infty}^{0} + \left(\frac{-1}{1+j2\pi f}\right) e^{-(1+j2\pi f)t}\Big|_{0}^{+\infty} =$$

$$= \frac{1}{1-j2\pi f} + \frac{1}{1+j2\pi f} = \frac{(1+j2\pi f)+(1-j2\pi f)}{1+4\pi^2 f^2(-j^2)} = \frac{2}{1+4\pi^2 f^2}$$

$$x(t) = e^{-|t|} \Leftrightarrow X(f) = \frac{2}{1+4\pi^2 f^2}$$

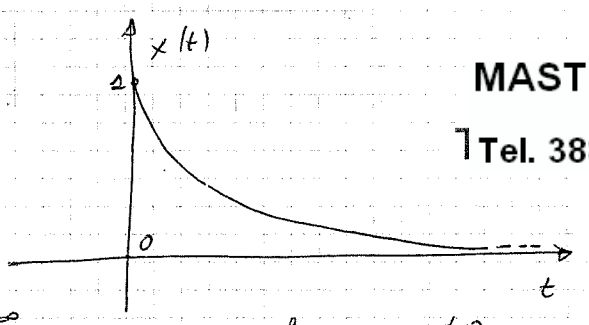
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Es 3 Calcolare la TCF esponenziale unilatera

$$x(t) = e^{-t} \cdot u(t)$$



$$X(f) = \int_{-\infty}^{+\infty} e^{-t} u(t)\, e^{-j2\pi f t}\, dt = \int_{0}^{+\infty} e^{-t} e^{-j2\pi f t}\, dt = \int_{0}^{+\infty} e^{-t(1+j2\pi f)}\, dt =$$

$$= -\frac{1}{1+j2\pi f} e^{-(1+j2\pi f)t}\Big|_0^{+\infty} =$$

$$= -\frac{1}{1+j2\pi f}$$

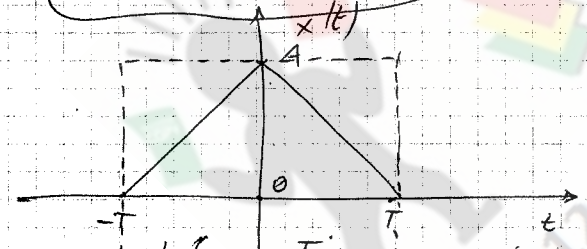
$$x(t) = e^{-t} u(t) \Leftrightarrow X(f) = -\frac{1}{1+j2\pi f}$$

Es 4 Calcolare la TCF triangolo

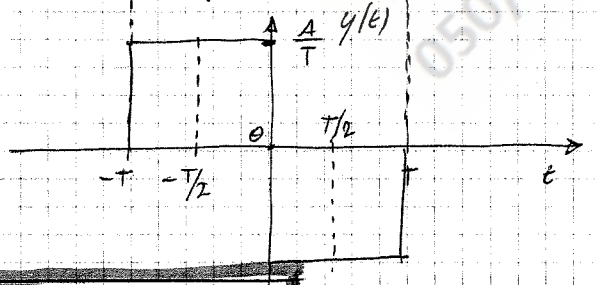
$$x(t) = A\left(1 - \frac{|t|}{T}\right) \text{rect}\left(\frac{t}{2T}\right)$$

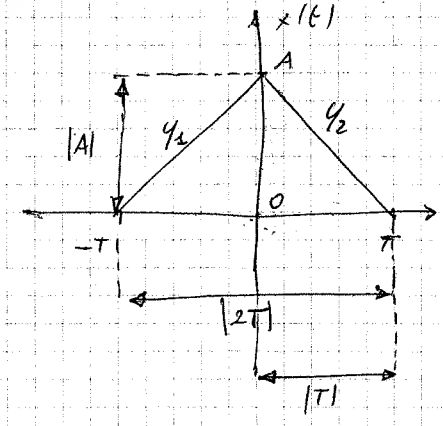
applicando il th della derivazione.

$$y(t) = \frac{d}{dt} x(t) \Rightarrow x(t) = \int_{-\infty}^{t} y(\alpha)\, d\alpha$$



$$y(t) = \frac{A}{T} \text{rect}\left(\frac{t+\frac{T}{2}}{T}\right) - \frac{A}{T} \text{rect}\left(\frac{t-\frac{T}{2}}{T}\right)$$





$$x(t) = A\left(1 - \frac{|t|}{T}\right) \text{rect}\left(\frac{t}{2T}\right)$$

$$\Updownarrow$$

$$X(f) = AT\, \text{sinc}^2(fT)$$

$$A\left(1 - \frac{|t|}{T}\right) = A \pm \frac{At}{T}$$

considerare il triangolo come due rette che passano nel punto A, una con coeff angolare positivo, l'altra negativo (|t|)

$$y_{1,2} = |m|x + q$$

$$y_1 = \frac{A}{T} t + A$$

$$y_2 = -\frac{A}{T} t + A$$

$$\Rightarrow |t| \frac{A}{T} + A$$

$$Y(f) = \left(\frac{A}{T}\right) T \operatorname{sinc}(fT) e^{-j2\pi f \frac{T}{2}} - \left(\frac{A}{T}\right) T \operatorname{sinc}(Tf) e^{-j2\pi f \frac{T}{2}} =$$

$$= \frac{2j}{A} \operatorname{sinc}(fT) \left[\frac{e^{j\pi fT} - e^{-j\pi fT}}{2j}\right] =$$

$$= 2j A \operatorname{sinc}(fT) \left[\sin(\pi fT)\right]$$

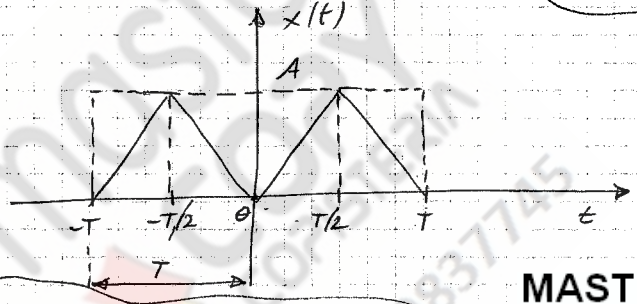
← equivalente ad un sin(x) (vedi formule di Eulero)

$$Y(f) = 2j A \operatorname{sinc}(fT) \left[\sin(\pi fT)\right]$$

$$X(f) = \frac{1}{j2\pi f} Y(f) = \frac{1}{j2\pi f} 2j A \operatorname{sinc}(fT) \frac{\sin(\pi fT)}{\pi f} \left(\frac{T}{T}\right) =$$

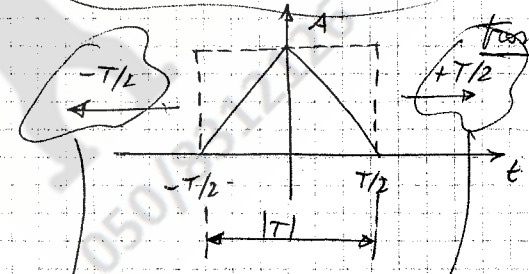
$$= AT \operatorname{sinc}(fT) \operatorname{sinc}(fT) = AT \operatorname{sinc}^2(fT)$$

EX5 Calcolare la TCF di due triangoli



$$y(t) = A\left(1 - \frac{|t|}{T/2}\right) \operatorname{rect}\left(\frac{t}{T}\right)$$

$$\Rightarrow X(f) = \frac{Y(f)}{j2\pi f}$$



traslazione / ritardo di:

(applicazione del th del ritardo) $A\left(1 - \frac{|t|}{\frac{T}{2}}\right) = A - \frac{A|t|}{\frac{T}{2}}$

$$x(t) = y\left(t - \frac{T}{2}\right) + y\left(t + \frac{T}{2}\right)$$

$$X(f) = Y(f) e^{-j2\pi f\left(\frac{T}{2}\right)} + Y(f) e^{j2\pi f\left(\frac{T}{2}\right)} = 2Y(f) \left[\frac{e^{-j\pi fT} + e^{j\pi fT}}{2}\right] =$$

$$= 2Y(f) \cos(\pi fT)$$

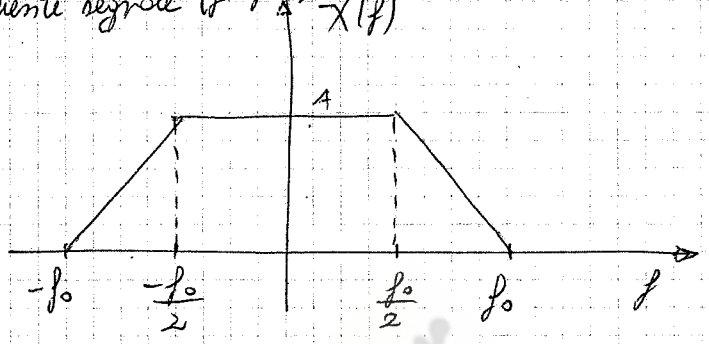
$Y(f) = AT' \operatorname{sinc}^2(fT')$ $\left(T' = \frac{T}{2}\right)$ con e quindi $Y(f) = A\frac{T}{2} \operatorname{sinc}^2\left(f\frac{T}{2}\right)$

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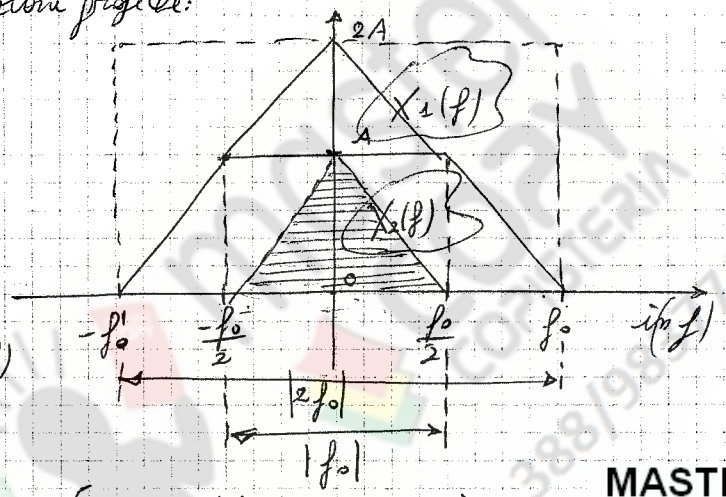
$$Y(f) = A\frac{T}{2}\,\mathrm{sinc}^2\left(f\frac{T}{2}\right)$$

$$X(f) = 2Y(f)\cos(\pi f T) = AT\,\mathrm{sinc}^2\left(f\frac{T}{2}\right)\left[\cos(\pi f T)\right]$$

ESERCIZIO Calcolare la (modulo della) ATCF (FT^{-1}) del seguente segnale (grafico) $X(f)$



1) soluzione considerazioni grafiche:



$$X(f) = X_1(f) - X_2(f)$$

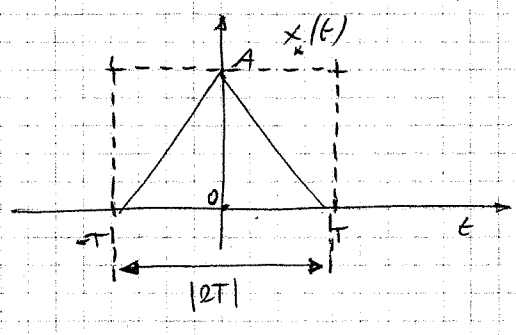
$$X_1(f) = 2A\left(1 - \frac{|f_{a1}|}{f_0}\right)\mathrm{rect}\left(\frac{f}{2f_0}\right)$$

$$X_2(f) = A\left(1 - \frac{|f|}{\frac{f_0}{2}}\right)\cdot\mathrm{rect}\left(\frac{f}{f_0}\right)$$

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so che dall'esercizio precedente $x_*(t) = A\left(1 - \frac{|t|}{T}\right)\mathrm{rect}\left(\frac{t}{2T}\right) \Leftrightarrow X_*(f) = AT\,\mathrm{sinc}^2(fT)$

grafici:



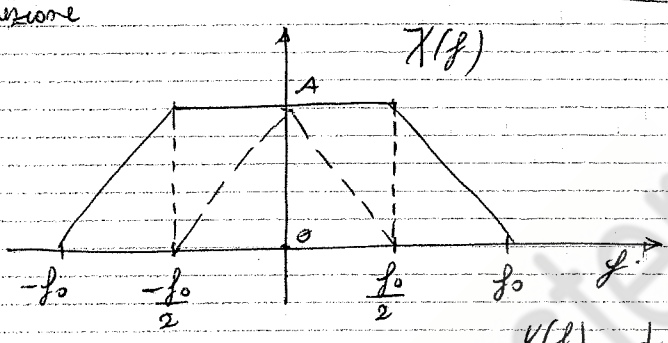
$X_1(f) \Leftrightarrow x_1(t) = +2A f_0 \operatorname{sinc}^2(t f_0)$

$X_2(f) \Leftrightarrow x_2(t) = A\left(\frac{f_0}{2}\right) \operatorname{sinc}^2\left(t \frac{f_0}{2}\right)$

$x(t) = x_1(t) + x_2(t) = 2A f_0 \operatorname{sinc}^2(f_0 t) + A\frac{f_0}{2} \operatorname{sinc}^2\left(\frac{f_0 t}{2}\right) =$

$= A f_0 \left[2 \operatorname{sinc}^2(f_0 t) + \frac{1}{2} \operatorname{sinc}^2\left(\frac{f_0 t}{2}\right) \right]$ 1 soluzione

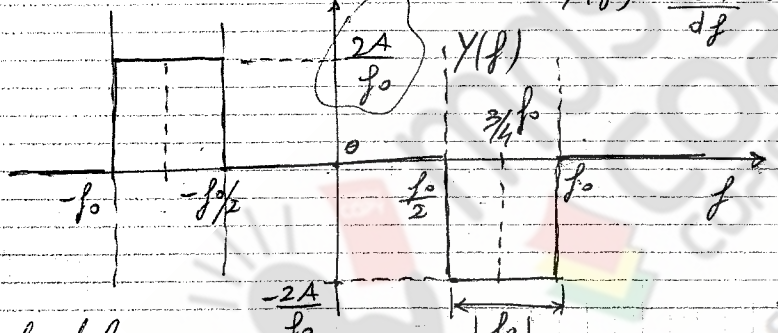
(2) soluzione



con applicazione del th della derivazione.

$Y(f) = \frac{d X(f)}{df}$ $\Rightarrow X(f) = \int_{-\infty}^{f} Y(\alpha)\, d\alpha$

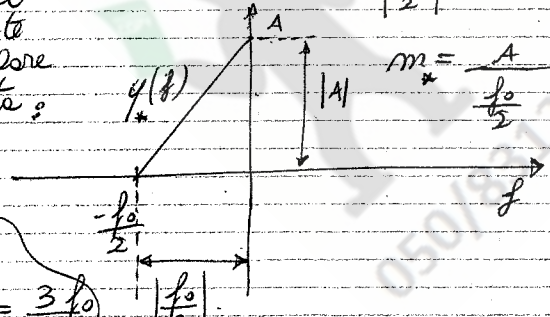
$\Rightarrow x(t) = \frac{-1}{j2\pi t} y(t)$



NOTA:

$y_*(f)$ è una retta

calcolo del coefficiente angolare della retta:



$m_* = \frac{A}{\frac{f_0}{2}} = \frac{2A}{f_0}$

$x_*(f) = \frac{d\, y_*(f)}{df}$ è il coeff. angolare della retta $y_*(f)$

$y_*(f) = m_* f + q$

$\frac{f_0}{2} + \frac{1}{2}\left(\frac{f_0}{2}\right) = \frac{3}{4} f_0$

$Y(f) = \left(\frac{2A}{f_0}\right) \operatorname{rect}\left(\frac{f + \frac{3}{4} f_0}{f_0/2}\right) - \left(\frac{2A}{f_0}\right) \operatorname{rect}\left(\frac{f + \frac{3}{4} f_0}{\frac{f_0}{2}}\right)$

NOTA:

$f_0 - \frac{f_0}{2} = \frac{2f_0 - f_0}{2} = \frac{1}{2} f_0$

$\frac{f_0}{2} + \frac{f_0}{4} = \frac{2f_0 + f_0}{4} = \frac{3}{4} f_0$

